spherical segment and its supporting plane without communicating to them a high polish.

M. Hansen remarks, that the form of construction here proposed by him especially possesses the advantage that, when the oscillations lie in one plane, or nearly so, a rotation of the pendulum about its axis (which he had shown to arise readily from the azimuthal motion of the plane of oscillation depending on the diurnal rotation of the earth) is no longer possible. The remainder of the section is occupied with an investigation of the motion of such a pendulum.

On the Secular Variation of the Moon's Mean Motion.

By J. C. Adams, Esq., M.A. F.R.S. &c.*

The author remarks, that in treating a great problem of approximation, such as that presented to us by the investigation of the moon's motion, experience shows that nothing is more easy than to neglect, on account of their apparent insignificance, considerations which ultimately prove to be of the greatest importance. One instance of this occurs with reference to the secular acceleration of the moon's mean motion. Although this acceleration and the diminution of the eccentricity of the earth's orbit, on which it depends, had been made known by observation as separate facts, yet many of the first geometers altogether failed to trace any connexion between them, and it was not until he had made repeated attempts to explain the phenomenon by other means, that Laplace himself succeeded in referring it to its true cause.

The accurate determination of the amount of the acceleration is a matter of very great importance. The effect on the moon's place, of an error in any of the periodic inequalities, is always confined within certain limits, and takes place alternately in opposite directions within very moderate intervals of time, whereas the effect of an error in the acceleration goes on increasing for an almost indefinite period, so as to render it impossible to connect observations made at very distant times.

In the *Mécanique Céleste*, the approximation to the value of the acceleration is confined to the principal term, but in the theories of Damoiseau and Plana, the developements are carried to an immense extent, particularly in the latter, where the multiplier of the change in the square of the eccentricity of the earth's orbit, which occurs in the expression of the secular acceleration, is given to terms of the seventh order.

As these theories agree in principle, and only differ slightly in the numerical value which they assign to the acceleration, and as they passed under the examination of Laplace, with especial reference to this subject, it might be supposed that only some small numerical rectifications would be required in order to obtain a very exact determination of this value.

It has not been, therefore, without surprise, which he has no doubt will be shared by the Society, that the author has lately

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* Proc. of the Roy. Soc. June 16, 1853.

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found that Laplace's explanation of the phenomenon in question is essentially incomplete, and that the numerical results of Damoiseau's and Plana's theories, with reference to it, consequently require to be very sensibly altered.

Laplace's explanation may be briefly stated as follows. He shows that the mean central disturbing force of the sun, by which the moon's gravity towards the earth is diminished, depends not only on the sun's mean distance, but also on the eccentricity of the earth's orbit. Now this eccentricity is at present (and for many ages has been) diminishing, while the mean distance remains unaltered. In consequence of this, the mean disturbing force is also diminishing, and therefore the moon's gravity towards the earth at a given distance is, on the whole, increasing. Also the area described in a given time by the moon about the earth is not affected by this alteration of the central force; whence it readily follows that the moon's mean distance from the earth will be diminished in the same ratio as the force at a given distance is increased, and the mean angular motion will be increased in double the same ratio.

This, the author states, is the main principle of Laplace's analytical method, in which he is followed by Damoiseau and Plana; but it will be observed that this reasoning supposes that the area described by the moon in a given time is not permanently altered, or, in other words, that the tangential disturbing force produces no permanent effect. On examination, however, he remarks it will be found that this is not strictly true, and he proceeds briefly to point out the manner in which the inequalities of the moon's motion are modified by a gradual change of the disturbing force, so as to give rise to such an alteration of the areal velocity.

As an example, he takes the case of the *variation*, the most direct effect of the disturbing force. In the ordinary theory, the orbit of the moon, as affected by this inequality only, would be symmetrical with respect to the line of conjunction with the sun, and the areal velocity generated while the moon was moving from quadrature to syzygy, would be exactly destroyed while it was moving from syzygy to quadrature, so that no permanent alteration would be produced.

In reality, however, the magnitude of the disturbing force by which this inequality is caused, depends in some degree on the eccentricity of the earth's orbit; and as this is continually diminishing, the disturbing forces at equal intervals before and after conjunction will not be exactly equal. Hence the orbit will no longer be symmetrically situated with respect to the line of conjunction, and therefore the effects of the tangential force before and after conjunction no longer exactly balance each other.

The other inequalities of the moon's motion will be similarly modified, especially those which depend, more directly, on the eccentricity of the earth's orbit, so that each of them will give rise to an uncompensated change of the areal velocity, and all of these must be combined in order to ascertain the total effect.

Since the distortion of the orbit just pointed out is due to the

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change of the disturbing force consequent upon a change in the eccentricity of the earth's orbit, and the action of the tangential force permanently to change the rate of description of areas, is only brought into play by means of this distortion, it follows that the alteration of the areal velocity will be of the order of the square of the disturbing force multiplied by the rate of change of the square of the eccentricity. It is evident that this alteration of areal velocity will have a direct effect in changing the acceleration of the moon's mean motion.

Having thus briefly indicated the way in which the effect now treated of originates, the author proceeds with the analytical investigation of its amount. In the present communication, however, he proposes to confine his attention to the principal term of the change thus produced in the acceleration of the moon's mean motion, deferring to another, though he hopes not a distant opportunity, the fuller treatment of this subject, as well as the determination of the secular variations of the other elements of the moon's motion, which, arising from the same cause, have also been hitherto overlooked.

In the usual theory, the reciprocal of the moon's radius vector is expressed by means of a series of *cosines* of angles formed by combinations of multiples of the mean angular distance of the moon from the sun, of the mean anomalies of the moon and sun, and of the moon's mean distance from the node; and the moon's longitude is expressed by means of a series of *sines* of the same angles, the coefficients of the periodic terms being functions of the ratio of the sun's mean motion to that of the moon, of the eccentricities of the two orbits and of their mutual inclination.

Now, if the eccentricity of the earth's orbit be supposed to remain constant, this is the true form of the expressions for the moon's co-ordinates; but if that eccentricity be variable, the author shows that the differential equations cannot be satisfied without adding to the expression for the reciprocal of the radius vector, a series of small supplementary terms depending on the *sines* of the angles whose *cosines* are already involved in it, and to the expression for the longitude, a series of similar terms depending on the *cosines* of the same angles; all the coefficients of these new terms containing as a factor the differential coefficient of the eccentricity of the earth's orbit taken with respect to the time.

The author first determines as many of these terms as are necessary in the order of approximation to which he restricts himself, and then takes them into account in the investigation of the secular acceleration. The expression which he thus obtains for the first two terms of this acceleration, is,

$$-\left(\frac{3}{2}m^2-\frac{3771}{64}m^4\right)\int(e'^2-\mathrm{E}'^2)\,n\,dt.$$

According to Plana, the corresponding expression is

$$-\left(\frac{3}{2}m^2-\frac{2187}{128}m^4\right)\int(e'^2-\mathrm{E}'^2)\,n\,dt.$$

It will be observed that the coefficient of the second term has been

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completely altered in consequence of the introduction of the new terms.

The numerical effect of this alteration is to diminish by 1″·66 the coefficient of the square of the time in the expression for the secular acceleration; the time being, as usual, expressed in centuries.

It will, of course, be necessary to carry the approximation much further, in order to obtain such a value of this coefficient as may be employed with confidence in the calculation of ancient eclipses.

Considerations of a General Nature relative to the small Planets situate between Mars and Jupiter. By M. Le Verrier.*

M. Le Verrier commences this brief communication by remarking that the hypothesis of Olbers respecting the origin of the small planets, which the great inclination of the orbit of *Pallas* did not serve to confirm, has been completely negatived by the numerous discoveries of recent years. It is to be hoped, he adds, that, since the hypothesis of Olbers must be renounced, the knowledge of a great number of such bodies may finally lead to the discovery of some law regulating their distribution in space. He then proceeds to some general considerations respecting the twenty-six planets already discovered, omitting all reference to the twenty-seventh planet which had been recently discovered by Mr. Hind.

The small planets revolve in a zone which commences at the mean distance of 2·20 from the sun, and extends to the distance 3·16. The mean value of their excentricities is 0·155. The magnitude of the excentricity in individual cases does not seem to have any relation either to the mean distance from the sun or to the position of the perihelion.

The inclinations of the orbits, whether with respect to each other or to the ecliptic, are in both cases considerable. The individual magnitude of the inclinations does not appear to depend either on the mean distance from the sun or on the direction of the ascending node.

The perihelia and the ascending nodes offer some peculiar features. *Twenty* of the perihelia have their longitudes included between 4° and 184°. *Twenty-two* of the ascending nodes have their longitudes between 36° and 216°. M. Le Verrier appears somewhat inclined to suspect the existence of a systematic difference between the mean direction of the ascending nodes of the planets nearer the sun and that of the ascending node of the planets that are more remote, indicating the existence of two distinct groups. He then proceeds to investigate the superior limit of the total quantity of matter which may be distributed in the zone of the heavens under consideration.

An inquiry of this nature can only be conducted by an exa-

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* *Comptes Rendus*, Nov. 28, 1853.